UNIT-I

Plane

⇒ point (x, y, z)

$A(x_1, y_1, z_1)$

$B(x_2, y_2, z_2)$

⇒ Distance

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

⇒ Direction ratios

$a = x_2 - x_1$, $b = y_2 - y_1$, $c = z_2 - z_1$

⇒ Direction cosines

$a = x_2 - x_1$, $b = y_2 - y_1$, $c = z_2 - z_1$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad M = \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

practice questions

1. Distance:-

$A = (-1, 0, 2)$

$B = (1, 0, -2)$

$$AB = \sqrt{(1-(-1))^2+(0-0)^2+(-2-2)^2}$$
$$= \sqrt{(1+1)^2+0+(-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= \sqrt{4 \times 5}$$
$$= 2\sqrt{5}$$

Direction ratios :-

$a = 1-(-1)$, $b = 0-0$, $c = -2-2$

$a = 2$, $b = 0$, $c = -4$

Direction cosines :-

$a = x_2 - x_1$, $b = y_2 - y_1$, $c = z_2 - z_1$

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$l = \frac{2}{\sqrt{4+0+16}}, \quad m = \frac{0}{\sqrt{4+0+16}}, \quad n = \frac{-4}{\sqrt{4+0+16}}$$

$$l = \frac{2}{\sqrt{20}}, \quad m = \frac{0}{\sqrt{20}}, \quad n = \frac{-4}{\sqrt{20}}$$

$$l = \frac{2}{2\sqrt{5}}, \quad m = \frac{0}{2\sqrt{5}}, \quad n = \frac{-4}{2\sqrt{5}}$$

$$l = \frac{1}{\sqrt{5}}, \quad m = 0, \quad n = \frac{-2}{\sqrt{5}}$$

Concept

~~L_1,~~

$L_1 : a_1, b_1, c_1$

$k \neq 0$, ka_1, kb_1, kc_1

$L_2 : a_2, b_2, c_2$

Angle between L_1 and L_2

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$\Rightarrow$ $\bar{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\bar{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos(\bar{A},\bar{B})$

$a_1a_2 + b_1b_2 + c_1c_2 = \sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}\cos\theta$

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$L_1 \perp L_2 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$ Perp.

$L_1 \parallel L_2 \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\Rightarrow$ cross product

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$|\bar{A}||\bar{B}| \sin\theta \, \hat{n} = \hat{i}(b_1c_2 - b_2c_1) - \hat{j}(a_1c_2 - a_2c_1) + \hat{k}(a_1b_2 - a_2b_1)$$

Plane

Plane :-

$ax + by + cz + d = 0$, $\sqrt{a^2+b^2+c^2} \neq 0$

$A(x_1, y_1, z_1)$

$B(x_2, y_2, z_2)$

$C(x_3, y_3 - z_3)$

Let A, B, C be three no collinear points then there will be unique plane passing through A, B, C

Example for only understanding

$ax + by + cz + d = 0$
$ax_1 + by_1 + cz_1 + d = 0$
$ax_2 + by_2 + cz_2 + d = 0$
$ax_3 + by_3 + cz_3 + d = 0$

$$\begin{bmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

(or)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Problem: Find the plane passing through $A(1,0,0)$, $B(1,2,0)$, $C(1,2,3)$.

Sol:- The plane is

$$\begin{vmatrix} x-1 & y-0 & z-0 \\ 1-0 & 2-0 & 0-0 \\ 1-1 & 2-0 & 3-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y & z \\ 0 & 2 & 0 \\ 0 & 2 & 3 \end{vmatrix} = 0$$

$$x-1(6-0) - y(0-0) + z(0-0) = 0$$

$$6(x-1) = 0 \Rightarrow x-1 = \frac{0}{6} \Rightarrow x-1 = 0$$

problem : Find the plane passing through $A(1,0,0)$, $B(0,2,0)$, $C(0,0,3)$

Sol:- The plane is

$$\begin{vmatrix} x-1 & y-0 & z-0 \\ 0-1 & 2-0 & 0-0 \\ 0-1 & 0-0 & 3-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y & z \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 0$$

25-06

$$(x-1)(6-0) - y(-3) + z(+2) = 0$$

$$6x - 6 + 3y + 2z = 0$$

$$6x + 3y + 2z - 6 = 0$$

problem : Find the plane passing through $A(1,2,3)$, $B(-1,-2,-3)$, $C(0,1,5)$

Sol:- The plane is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ -1-1 & -2-2 & -3-3 \\ 0-1 & 1-2 & 5-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ -2 & -4 & -6 \\ -1 & -1 & 2 \end{vmatrix} = 0$$

$$(x-1)(-8-6) - (y-2) + (z-3)(2-4) = 0$$

$$(x-1)(-14) - (y-2)(-10) + (z-3)(-2) = 0$$

$$-14x + 14 + 10y - 20 - 2z + 6 = 0$$

$$-14x + 10y - 2z = 0$$

$$2(-7x + 5y - z) = 0$$

$$-7x + 5y - z = 0$$

$$7x - 5y + z = 0$$

Chaking Rough

$ax+by+cz=0$

A point on checking

$1(-7)+2(5)+3(-1)=0$

$-7+10-3=0$

$-10+10=0$

B point on checking

$-1(-7)+(-2)(5)+(-3)(-1)=0$

$7-10+3=0$

$10-10=0$

C point on checking

$0(-7)+1(5)+5(-1)=0$

$0+5-5=0$

$5-5=0$

25-06-2022

⇒ Distance between two planes π_1 and π_2

$$\frac{|k_1-k_2|}{\sqrt{a^2+b^2+c^2}}$$

Rough

$$\frac{|20-3|}{\sqrt{16+36+100}} = \frac{17}{\sqrt{152}}$$

Ex:- π_1 : $ax+by+cz+k_1=0$

π_2 : $ax+by+cz+k_2=0$

$2x+3y+5z+10=0 \times 2$

$4x+6y+10z+3=0 \times 1$

$4x+6y+10z+10=0$

$4x+6y+10z+3=0$

⇒ $A(x_1,y_1,z_1)$ the perpendicular distance from A to the plane $ax+by+cz+d=0$

is $$\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

⇒ $ax^2+by^2+cz^2+2hxy+2fyz+2gzx+dx+ey+kz+g=0$

$ax^2+by^2+cz^2+2hxy+2fyz+2gzx=0$

$ax^2+by^2+cz^2+2hxy+2fyz+2gzx=0$

$ax^2+(2hy+2gz)x+by^2+cz^2+2fyz=0$

$Ax^2+Bx+C=0$

$A=a \quad , \quad B=2hy+2gz \quad , \quad C=by^2+cz^2+2fyz$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$l_1x+m_1y+n_1z=0$

$l_2x+m_2y+n_2z=0$

$d_1d_2=g$

$l_1d_2+l_2d_1=d$

$m_1d_2+m_2d_1=e$

1. Show that the equation $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents pair of planes. Also find the angle between them.

Sol:- $2x^2+(2z+y)x+(-6y^2-12z^2+18yz)=0$

$A=2, \quad B=2z+y, \quad C=(-6y^2-12z^2+18yz)$

$B^2-4AC=(2z+y)^2-4(2)(-6y^2-12z^2+18yz)$

$=4z^2+y^2+4yz+48y^2+96z^2-144yz$

$=100z^2+49y^2-140yz$

$=(10z)^2+(7y)^2-2(10z)(7y)$

$B^2-4AC=(7y-10z)^2$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$=\frac{-(2z+y)\pm\sqrt{(7y-10z)^2}}{2(2)}$$

$$= \frac{-2z - y \pm (7y - 10z)}{1 \cdot 4}$$

$$x = \frac{-2z - y + 7y - 10z}{4}$$

$$4x = 6y - 12z$$

$$4x - 6y + 12z = 0$$

$$2x - 3y + 6z = 0 \rightarrow (1)$$

$$x = \frac{-2z - y - (7y - 10z)}{4}$$

$$4x = -2z - y - 7y + 10z$$

$$= -8y + 8z$$

$$4x + 8y - 8z = 0$$

$$x + 2y - 2z = 0 \rightarrow (2)$$

The required planes are

$$\underset{a_1}{2x} - \underset{b_1}{3y} + \underset{c_1}{6z} = 0 \quad , \quad \underset{a_2}{x} + \underset{b_2}{2y} - \underset{c_2}{2z} = 0$$

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{|2 \cdot 1 + (-3 \cdot 2) + (6 \cdot -2)|}{\sqrt{2^2 + (-3)^2 + 6^2}\sqrt{1^2 + 2^2 + (-2)^2}}$$

$$\cos\theta = \frac{|2 - 6 - 12|}{\sqrt{4+9+36}\sqrt{1+4+4}} = \frac{|-16|}{\sqrt{49}\sqrt{9}}$$

$$\cos\theta = \frac{16}{7(3)} = \frac{16}{21}$$

$$\theta = \cos^{-1}\frac{16}{21} \quad //$$

27-06-2022

2. Find the planes represented by the equation $x^2-2y^2-z^2-xy+3yz-6x+3y+9=0$ and hence find the angle between them.

Sol: $x^2-2y^2-z^2-xy+3yz=0$

$x^2+(-y)x+3yz-2y^2-z^2=0$

$A=1,\quad B=-y,\quad C=3yz-2y^2-z^2$

$B^2-4AC=(-y)^2-4(1)(3yz-2y^2-z^2)$

$=y^2-12yz+8y^2+4z^2$

$=9y^2+4z^2-12yz$

$=(3y)^2+(2z)^2-2(3y)(2z)$

$=(3y-2z)^2$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-(-y)\pm\sqrt{(3y-2z)^2}}{2(1)}$$

$$x=\frac{y\pm(3y-2z)}{2}$$

$2x=y\pm(3y-2z$

$2x=y+(3y-2z)$ or $2x=y-(3y-2z)$

$=y+3y-2z$ $\qquad$ $=y-3y+2z$

$=4y-2z$ $\qquad$ $=-2y+2z$

$2x-4y+2z=0$ $\qquad$ $2x+2y-2z=0$

$x-2y+z=0$ $\qquad$ $x+y-z=0$

Let the required planes are

$x-2y+z=0$ and $x+y-z=0$

then $(x-2y+z+d_1)(x+y-z+d_2) = x^2-2y^2-z^2-xy+3yz-6x+3y+9$

$x^2+xy-xz+xd_2$
$+(-2xy)+(-2y^2)+2yz+(-2yd_2)$
$+zx+zy-z^2+zd_2$
$+d_1x+d_1y-d_1z+d_1d_2 = x^2-2y^2-z^2-xy+3yz-6x+3y+9$

$x^2-2y^2-z^2-xy+3yz+(d_1+d_2)x+(-2d_2+d_1)y+(-d_1+d_2)z$
$+d_1d_2 = x^2-2y^2-z^2-xy+3yz-6x+3y+9$

$$\left.\begin{aligned} d_1+d_2 &= -6 \\ d_1-2d_2 &= 3 \end{aligned}\right\} \quad 3d_2 = -9 \Rightarrow d_2 = -3$$

$-d_1+d_2 = 0$ $\quad d_1 = -6-d_2 = -6-(-3)$

$d_1d_2 = 9$ $\quad = -6+3$
$\quad = -3$

$x-2y+z-3=0$
$x+y+z-3=0$

angle between two planes

$a_1 = 1, \ b_1 = -2, \ c_1 = 1$
$a_2 = 1, \ b_2 = 1, \ c_2 = -1$

$$\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{|(1\cdot1)+(-2\cdot1)+(1\cdot-1)|}{\sqrt{1^2+(-2)^2+1^2}\sqrt{1^2+(-1)^2+(-1)^2}}$$

$$= \frac{|1-2-1|}{\sqrt{1+4+1}\sqrt{1+1+1}}$$

$$= \frac{|-2|}{\sqrt{6}\sqrt{3}} = \frac{2}{\sqrt{18}}$$

$$= \frac{2}{\sqrt{9}\sqrt{2}}$$

$$= \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$ //

3. Show that $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$ represents a pair of parallel planes and find the distance between them.

Sol:- $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$

$x^2+4y^2+9z^2-12yz-6zx+4xy = 0$

$A=1$, $B=4y-6z$, $C=4y^2+9z^2-12yz$

$B^2-4AC = (4y-6z)^2-4(1)(4y^2+9z^2-12yz)$

$= 16y^2+36z^2-48yz-16y^2-36z^2+48yz$

$=0$

$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$= \frac{-4y-6z\pm\sqrt{0}}{2(1)}$$

$2x = -4y-6z$

$2x+4y-6z=0$

Let the required planes are

$2x+4y-6z+d_1=0$ or $2x+4y-6z+d_2=0$

$x+2y-3z+d_1=0$ or $x+2y-3z+d_2=0$

then

$(x+2y-3z+d_1)(x+2y-3z+d_2) = x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y+10y-15z+6$

$x^2+2xy-3xz+xd_2+2xy+4y^2-6yz+2yd_2-3zx-6yz+9z^2-3zd_2+xd_1+2yd_1-3zd_1+d_1d_2 = x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6$

$$x^2+4y^2+9z^2-12yz-6zx+4xy+(d_1+d_2)x+(2d_1+2d_2)y-(3d_2+3d_1)z+d_1d_2=x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6$$

$d_1+d_2=5$

$2d_1+2d_2=10$

$-3d_1+3d_2=-15$

$d_1d_2=6$

$d_1=5-d_2$

$d_1d_2=6$

$(5-d_2)d_2=6$

$5d_2-d_2^2=6$

$5d_2-d_2^2-6=0$

$-5d_2+d_2^2+6=0$

$d_2^2-5d_2+6=0$

$6d_2^2$ → $-3d_2$, $-2d_2$

$d_2^2-3d_2-2d_2+6=0$

$d_2(d_2-3)-2(d_2-3)=0$

$d_2-2=0$, $d_2-3=0$

$d_2=2$, $d_2=3$

checking:

$\Rightarrow d_1d_2=6 \Rightarrow d_1(3)=6$

$d_1=\frac{6}{3}$

$d_1=2$

$\Rightarrow d_1+d_2=5 \Rightarrow d_1+3=5$

$d_1=5-3$

$d_1=2$

$\Rightarrow 2d_1+2(3)=10$

$2d_1+6=10$

$2d_1=10-6$

$d_1=4/2$

$d_1=2$

$\Rightarrow d_1+d_2=5 \Rightarrow 2+d_2=5$

$d_2=5-2$

$d_2=3$

$\Rightarrow 2d_1+2d_2=10 \Rightarrow 2(2)+2d_2=10$

$4+2d_2=10$

$2d_2=10-4$

$d_2=\frac{6}{2} \Rightarrow d_3=3$

The planes are

$x+2y-3z+2=0,\ x+2y-3z+3=0$

$$\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{|(1\cdot1)+(2\cdot2)+(-3\cdot-3)|}{\sqrt{1^2+2^2+(-3)^2}\sqrt{1^2+2^2+(-3)^2}}$$

$$= \frac{|1+4+9|}{\sqrt{1+4+9}\sqrt{1+4+9}}$$

$$= \frac{14}{\sqrt{14}\sqrt{14}}$$

$$= \frac{\sqrt{14}\sqrt{14}}{\sqrt{14}\sqrt{14}}$$

$$= 1$$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\theta = 0$

parallel planes

Distance parallel planes

$$\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|2-3|}{\sqrt{1^2+2^2+(-3)^2}}$$

$$= \frac{|-1|}{\sqrt{14}} = \frac{1}{\sqrt{14}}$$ //

Bisecting planes

$\pi_1 : a_1x + b_1y + c_1z + d_1 = 0$

$\pi_2 : a_2x + b_2y + c_2z + d_2 = 0$, $d_1, d_2 \geq 0$

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm \frac{(a_2x + b_2y + c_2z + d_2)}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

(1) The plane that bisects the region containing origin

$$\pi_3 : \frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

(2) other plane is

$$\pi_4 : \frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = -\left(\frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}\right)$$

acute angle $(\pi_1, \pi_2) = \theta < 90$

obtuse angle $(\pi_1, \pi_2) = \theta > 90$

If $\tan(\pi_1, \pi_3) < 1$ then π_3 is acute angle bisector, π_4 is obtuse angle bisector.

If $\tan(\pi_1, \pi_3) > 1$ then π_3 is obtuse angle bisector, π_4 is acute angle bisector.

1. Find the bisecting plane of acute angle between the planes $3x - 6y + 2z + 5 = 0$, $4x - 12y + 3z - 3 = 0$. Also find the plane bisecting the angle containing the origin.

Sol:- $\pi_1 : 3x - 6y + 2z + 5 = 0$,

$\pi_2 : 4x - 12y + 3z - 3 = 0$.

Bisecting planes are

$$\frac{3x-6y+2z+5}{\sqrt{3^2+(-6)^2+2^2}} = \pm\left(\frac{-4x+12y-3z+3}{\sqrt{(-4)^2+(12)^2+(-3)^2}}\right)$$

$$\frac{3x-6y+2z+5}{\sqrt{9+36+4}} = \pm\left(\frac{-4x+12y-3z+3}{\sqrt{16+144+9}}\right)$$

$$\frac{3x-6y+2z+5}{\sqrt{49}} = \pm\left(\frac{-4x+12y-3z+3}{\sqrt{169}}\right)$$

$$\frac{3x-6y+2z+5}{7} = \pm\left(\frac{-4x+12y-3z+3}{13}\right)$$

$$13(3x-6y+2z+5) = \pm(7(-4x+12y-3z+3))$$

$$39x-78y+26z+65 = \pm(-28x+84y-21z+21)$$

The plane that bisects the region containing origin is

$$39x-78y+26z+65 = -28x+84y-21z+21$$

$$39x-78y+26z+65+28x-84y+21z-21 = 0$$

$$\pi_3 : 67x-162y+47z+44 = 0$$

The other bisecting plane is

$$39x-78y+26z+65 = -(-28x+84y-21z+21)$$

$$39x-78y+26z+65 = +28x-84y+21z-21$$

$$39x-78y+26z+65-28x+84y-21z+21 = 0$$

$$\pi_4 : 11x+6y+5z+86 = 0$$

If θ is angle b/w π_1 and π_4 then

$$\cos\theta = \frac{|(3\cdot 11)+(-6\cdot 6)+(2\cdot 5)|}{\sqrt{3^2+(-6)^2+2^2}\sqrt{11^2+6^2+5^2}}$$

$$= \frac{|33-36+10|}{\sqrt{9+36+4}\sqrt{121+36+25}}$$

$$= \frac{7}{7\sqrt{182}} = \frac{1}{\sqrt{182}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(\frac{1}{182}\right)^2}$$

$$= \sqrt{1-\frac{1}{182}}$$

$$= \sqrt{\frac{182-1}{182}}$$

$$= \sqrt{\frac{181}{182}}$$

$$= \frac{\sqrt{181}}{\sqrt{182}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sqrt{181}/\sqrt{182}}{1/\sqrt{182}}$$

$$= \frac{\sqrt{181}}{\sqrt{182}} \times \frac{\sqrt{182}}{1}$$

$$\tan\theta = \sqrt{182} > 1$$

$$\theta > \frac{\pi}{4}$$

$\pi_4 : 11x+6y+5z+86=0$, bisects obtuse angle between π_1 and π_2

$\pi_3 : 67x-162y+47z+424=0$, bisects acute angle between π_1 and π_2

2\. A variable plane is at a constant distance 3p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the triangle ABC is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$

Sol:- Let the centroid of ΔABC is (x_1, y_1, z_1)

Since the required plane meets

x-axis at A $\Rightarrow A = (a,0,0)$

y-axis at B $\Rightarrow B = (0,b,0)$

z-axis at C $\Rightarrow C = (0,0,c)$

$\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}$

Centroid of $\Delta ABC = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$

$= \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$= (x_1, y_1, z_1)$

$\Rightarrow \frac{a}{3} = x_1 \Rightarrow a = 3x_1$

$\Rightarrow \frac{b}{3} = y_1 \Rightarrow b = 3y_1$

$\Rightarrow \frac{c}{3} = z_1 \Rightarrow c = 3z_1$

The required plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(or)

$\frac{x}{3x_1} + \frac{y}{3y_1} + \frac{z}{3z_1} = 1$

$\left(\frac{1}{3x_1}\right)x + \left(\frac{1}{3y_1}\right)y + \left(\frac{1}{3z_1}\right)z - 1 = 0$

The perpendicular distance from origin is 3p

3.

That is,

$$= \frac{|ax_1+by_1+cz_1|}{\sqrt{a^2+b^2+c^2}}$$

$$3p = \frac{\left|\left(\frac{1}{3x_1}\right)0 + \left(\frac{1}{3y_1}\right)0 + \left(\frac{1}{3z_1}\right)0 - 1\right|}{\sqrt{\left(\frac{1}{3x_1}\right)^2 + \left(\frac{1}{3y_1}\right)^2 + \left(\frac{1}{3z_1}\right)^2}}$$

$$\Rightarrow 3p = \frac{1}{\sqrt{\frac{1}{9x_1^2} + \frac{1}{9y_1^2} + \frac{1}{9z_1^2}}}$$

$$\Rightarrow 3p\sqrt{\frac{1}{9x_1^2} + \frac{1}{9y_1^2} + \frac{1}{9z_1^2}} = 1$$

$$\Rightarrow \sqrt{\frac{1}{9}\left(\frac{1}{x_1^2} + \frac{1}{y_1^2} + \frac{1}{z_1^2}\right)} = \frac{1}{3p}$$

$$\frac{1}{9}\left(\frac{1}{x_1^2} + \frac{1}{y_1^2} + \frac{1}{z_1^2}\right) = \frac{1}{9p^2}$$

$$x_1^{-2} + y_1^{-2} + z_1^{-2} = p^{-2}$$

The required locus is

$$x_0^{-2} + y_0^{-2} + z_0^{-2} = p^{-2}$$

3. If a plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r) then show that the equation of the plane $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$

Sol:- Let the centroid of $\triangle ABC$ is (p, q, r)

Since the required plane meets

x-axis at A => A = (a, 0, 0)

y-axis at B => B = (0, b, 0)

z-axis at C => C = (0, 0, c)

centroid of $\triangle ABC = \left(\frac{a+0+0}{3}, \frac{0+b+0}{3}, \frac{0+0+c}{3}\right)$

$= \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$= (p_a, q_b, r_c)$

$\Rightarrow \frac{a}{3} = p_a \Rightarrow a = 3p_a$

$\Rightarrow \frac{b}{3} = q_b \Rightarrow b = 3q_b$

$\Rightarrow \frac{c}{3} = r_c \Rightarrow c = 3r_c$

The required plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

(or)

$\frac{x}{3p_a} + \frac{y}{3q_b} + \frac{z}{3r_c} = 1$

$\frac{1}{3}\left(\frac{x}{p_a} + \frac{y}{q_b} + \frac{z}{r_c}\right) = 1$

$\frac{x}{p_a} + \frac{y}{q_b} + \frac{z}{r_c} = 3$ //

5m (4) Find the equation of the plane through the line of intersection of the plane $x+2y+3z+4=0$, $4x+3y+3z+1=0$ and perpendicular to $x+y+z+9=0$

Sol: π_1 :

π_2

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Sol: $\pi_1 : x+2y+3z+4=0$, $\pi_2 : 4x+3y+3z+1=0$

$\pi_3 : x+y+z+9=0$

The plane that pass through intersection of π_1 and π_2 is

$$\pi_1 + \lambda\pi_2 = 0$$

$(x+2y+3z+4)+\lambda(4x+3y+3z+1)=0$

$x+2y+3z+4+4\lambda x+3\lambda y+3\lambda z+\lambda=0$

$(1+4\lambda)x+(2+3\lambda)y+(3+3\lambda)z+4+\lambda=0$

If this plane is $\perp$ to π_3

then,

$(1)(1+4\lambda)+(1)(2+3\lambda)+(1)(3+3\lambda)=0$

$1+4\lambda+2+3\lambda+3+3\lambda=0$

$10\lambda+6=0$

$10\lambda=-6$

$\lambda=\frac{-6}{10}=\frac{-3}{5}$

The required plane is

$(1+4(-3/5))x+(2+3(-3/5))y+(3+3(-3/5))z+(4+(-3/5))=0$

$-7/5x+1/5y+6/5z+17/5=0$

$-7x+y+6z+17=0$

(or) $7x-y-6z-17=0$ //

Angle between two planes

1. Find the angles between the planes $2x-y+z=6$, $x+y+2z=7$

Sol If θ is an angle between the given planes, then

$$\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{|2(1)+(-1)(1)+(1)(2)|}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+1^2+2^2}}$$

$$= \frac{|2-1+2|}{\sqrt{4+1+1}\sqrt{1+1+4}}$$

$$= \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1} 1/2$$

$$\theta = 60^\circ$$

② Find the equation of the plane through the point $(1,0,-2)$ and $\perp$r to each of the planes $2x+y-z-2=0$ and $x-y-z-3=0$.

The given equation of the plane through the point is $(1,0,-2)$ is

$a(x-1)+b(y-0)+c(z-2)=0 \quad \text{---}\rightarrow ①$

If the plane ① is $\perp$r to

$2x+y-z-2=0$ then $2a+b-c=0 \longrightarrow$ ②

If the plane ① is ⊥r to

$x-y-z-3=0$ then $a-b-c=0 \longrightarrow$ ③

Now, the plane using ①, ② and ③

$$\begin{vmatrix} x-1 & y-0 & z-2 \\ 2 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$(x-1)(-1-1) - y(-2+1) + (z+2)(-2-1) = 0$

$(x-1)(-2) + 2y - y + (z+2)(-3) = 0$

$-2x + 2 + y + (-3z) + (-6) = 0$

$-2x + y - 3z - 4 = 0$

$2x - y + 3z + 4 = 0$

③ Find the equation of the plane through the point $(4,4,0)$ and ⊥r to each of the plane $x+2y+2z-5=0$ and $3x+3y+2z-8=0$.

The given equation of the plane through the point $(4,4,0)$ is

$a(x-4)+b(y-4)+c(z-0)=0$

$a(x-4)+b(y-4)+cz=0 \longrightarrow$ ①

If ① is ⊥r to the plane is

$x+2y+2z-5=0$ then $a+2b+2c=0 \longrightarrow$ ②

If ① is ⊥r to the plane is

$3x+3y+2z-8=0$ then $3a+3b+2c=0 \longrightarrow$ ③

Now, the plane using ①, ② and ③

$$\begin{vmatrix} x-4 & y-4 & z \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{vmatrix} = 0$$

$(x-4)(4-6) - (y-4)(2-6) + z(3-6) = 0$

$(x-4)(-2) - (y-4)(-4) + z(-3) = 0$

$-2x+8+4y-16-3z = 0$

$-2x+4y-3z-8 = 0$

$\therefore 2x-4y+3z+8=0$

$\therefore$ the equation of the required plane is

$2x-4y+3z+8=0$

(4) Find the equation of the plane through the point $(-1,3,2)$ and $\perp r$ to each of the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$

The given equation of the plane through the point $(-1,3,2)$ is

$a(x-(-1)) + b(y-3) + c(z-2) = 0$

$a(x+1)+b(y-3)+c(z-2) = 0$ —— (1)

If (1) is $\perp r$ to $x+2y+2z-5=0$ then

$a+2b+2z=0$ —— (2)

If (1) is $\perp r$ to the plane $3x+3y+2z-8=0$

then $3a+3b+2z=0$ —— (3)

Now the plane using (1), (2) and (3)

$$\begin{vmatrix} x+1 & y-3 & z-2 \\ 1 & 2 & 2 \\ 3 & 3 & 2 \end{vmatrix} = 0$$

$(x+1)(4-6) - (y-3)(2-6) + (z-2)(3-6) = 0$

$(x+1)(-2) + (-y+3)(-4) + (z-2)(-3) = 0$

$-2x - 2 + 4y - 12 - 3z + 6 = 0$

~~$-2x + 4y - 2 - 12 - 3z + 6 = 0$~~

$-2x + 4y - 3z - 14 + 6 = 0$

$-2x + 4y - 3z - 8 = 0$

$\therefore 2x - 4y + 3z + 8 = 0$

The required plane is $2x - 4y + 3z + 8 = 0$

(5) A variable plane is at a constant distance

6\. p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the tetrahedran OABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$

Let the centroid of the tetrahedran OABC is (x_1, y_1, z_1). Since the required plane meets ~~o-axi~~

O-axis at O $\Rightarrow$ O $(0,0,0)$

A-axis at A $\Rightarrow$ A $(a,0,0)$

B-axis at B $\Rightarrow$ B $(0,b,0)$

C-axis at C $\Rightarrow$ C $(0,0,c)$

$$\text{centroid tetrahedran} = \left(\frac{x_1+x_2+x_3+x_4}{4}, \frac{y_1+y_2+y_3+y_4}{4}, \frac{z_1+z_2+z_3+z_4}{4}\right)$$

$$= \left(\frac{0+a+0+0}{4}, \frac{0+0+b+0}{4}, \frac{0+0+0+c}{4}\right)$$

$$= \left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$$

$$= (x_1, y_1, z_1)$$

$\Rightarrow \frac{a}{4} = x_1 \Rightarrow a = 4x_1$

$\Rightarrow \frac{b}{4} = y_1 \Rightarrow b = 4y_1$

$\Rightarrow \frac{c}{4} = z_1 \Rightarrow c = 4z_1$

The required plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{4x_1} + \frac{y}{4y_1} + \frac{z}{4z_1} - 1 = 0$$

$$\left(\frac{1}{4x_1}\right)x + \left(\frac{1}{4y_1}\right)y + \left(\frac{1}{4z_1}\right)z - 1 = 0$$

The ⊥r distance from the origin is p

i.e.:- $p = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$

$$p = \frac{\left|\left(\frac{1}{4x_1}\right)0 + \left(\frac{1}{4y_1}\right)0 + \left(\frac{1}{4z_1}\right)0 - 1\right|}{\sqrt{\left(\frac{1}{4x_1}\right)^2 + \left(\frac{1}{4y_1}\right)^2 + \left(\frac{1}{4z_1}\right)^2}}$$

$$p = \frac{|-1|}{\sqrt{\frac{1}{16x_1^2} + \frac{1}{16y_1^2} + \frac{1}{16z_1^2}}}$$

$$P\sqrt{\frac{1}{16x_1^2}+\frac{1}{16y_1^2}+\frac{1}{16z_1^2}} = 1$$

$$\sqrt{\frac{1}{16x_1^2}+\frac{1}{16y_1^2}+\frac{1}{16z_1^2}} = \frac{1}{P}$$

squaring on both sides

$$\frac{1}{16x_1^2}+\frac{1}{16y_1^2}+\frac{1}{16z_1^2} = \frac{1}{P^2}$$

$$\frac{1}{16}\left(\frac{1}{x_1^2}+\frac{1}{y_1^2}+\frac{1}{z_1^2}\right) = \frac{1}{P^2}$$

$$x_1^{-2}+y^{-2}+z_1^{-2} = 16P^{-2}$$

The required locus is

$$\therefore x^{-2}+y^{-2}+z^{-2} = 16P^{-2}$$

Angle Bisector planes

(1) Find the equation of the acute angle bisector of the angle between the planes $3x-2y+6z+2=0$, $2x-y+2z+2=0$

The given planes are

$\pi_1 : 3x-2y+6z+2=0$

$\pi_2 : 2x-y+2z+2=0$

Bisecting planes are

$$\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = \pm \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\frac{3x-2y+6z+2}{\sqrt{3^2+(-2)^2+6^2}} = \pm \frac{2x-y+2z+2}{\sqrt{2^2+(-1)^2+2^2}}$$

$$\frac{3x-2y+6z+2}{\sqrt{9+4+36}} = \pm \frac{2x-y+2z+2}{\sqrt{4+1+4}}$$

$$\frac{3x-2y+6z+2}{\sqrt{49}} = \pm \frac{2x-y+2z+2}{\sqrt{9}}$$

$$\frac{3x-2y+6z+2}{7} = \pm \frac{2x-y+2z+2}{3}$$

$3(3x-2y+6z+2) = \pm(7(2x-y+2z+2))$

$9x-6y+18z+6 = \pm(14x-7y+14z+14)$

The plane that bisects the region containing origin is

$9x-6y+18z+6 = 14x-7y+14z+14$

$9x-6y+18z+6-14x+7y-14z-14=0$

$-5x+y+4z-8=0$

$\pi_3 : 5x-y-4z+8=0$

The other bisecting plane

$9x-6y+18z+6 = -(14x-7y+14z+14)$

$9x-6y+18z+6 = -14x+7y-14z-14$

$9x+14x-6y-7y+18z+14z+6+14=0$

$\pi_4 : 23x-13y+32z+20=0$

If θ is the angle between π_1 and π_3

$\pi_1 : 3x-2y+6z+2=0$

$\pi_3 : 5x-y-4z+8=0$

$$\cos\theta = \frac{|3(5)+(-2)(-1)+6(-4)|}{\sqrt{3^2+(-2)^2+6^2}\sqrt{5^2+(-1)^2+(-4)^2}}$$

$$= \frac{|15+2-24|}{\sqrt{9+4+36}\sqrt{25+1+16}}$$

$$= \frac{|-7|}{\sqrt{49}\sqrt{42}}$$

$$= \frac{7}{7\sqrt{42}}$$

$$\cos\theta = \frac{1}{\sqrt{42}}$$

Now, $\sin\theta = \sqrt{1-\cos^2\theta}$

$$= \sqrt{1-\left(\frac{1}{\sqrt{42}}\right)^2}$$

$$= \sqrt{1-\frac{1}{42}}$$

$$= \sqrt{\frac{42-1}{42}} = \sqrt{\frac{41}{42}} = \frac{\sqrt{41}}{\sqrt{42}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{\frac{\sqrt{41}}{\sqrt{42}}}{\frac{1}{\sqrt{42}}} = \sqrt{41}$$

$\tan\theta = \sqrt{41} > 1$

~~$\theta = \sqrt{41}$~~ $\theta > \frac{\pi}{4}$

$\pi_3 : 5x - y - 4z + 8 = 0$ bisects obtuse angle between π_1 and π_2

$\pi_4 : 23x - 13y + 32z + 20 = 0$ bisects acute angle between π_1 and π_2

2. A variable plane passes through a fixed point (α, β, γ) and meets the co-ordinate axes in A, B, C, show that the locus of the point of intersection of the plane through A, B, C parallel to the co-ordinate plane is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$.

Let the intercepts of a variable plane be a, b and c

Equation of the variable plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{---} (1)$$

A-axis at A => A(a, 0, 0)
B-axis at B => B(0, b, 0)
C-axis at C => C(0, 0, c)

Let $P(x_1, y_1, z_1)$ be the point of intersection of

Since this passes through (α, β, γ)

$$\frac{\alpha}{a} + \frac{\beta}{b} + \frac{\gamma}{c} = 1$$

The plane that passes through A and parallel to yz plane is $x = a$

The plane that passes through B and parallel to xz plane is $y = b$

The plane that passes through C and parallel to xy plane is $z = c$

Then $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$.

(3) Find the equation of the plane passing through (2, -3, 1) and whose normal is the joining the points (3, 4, -1) and (2, -1, 5).

Let $ax+by+cz+d=0$ be the required plane.

A (3, 4, -1) and B (2, -1, 5)

AB direction ratio's are

$a = x_2 - x_1$, $b = y_2 - y_1$, $c = z_2 - z_1$

$a = 2-3$, $b = -1-4$, $c = 5-(-1)$

$a = -1$, $b = -5$, $c = 6$

$-1x + (-5)y + 6z + d = 0$

$-x - 5y + 6z + d = 0$ — ①

Since the plane has to go through (2, -3, 1)

Substitute in equation ①

$-x - 5y + 6z + d = 0$

$-(2) - 5(-3) + 6(1) + d = 0$

$-2 + 15 + 6 + d = 0$

$19 + d = 0$

$d = -19$

$-x - 5y + 6z - 19 = 0$

$x + 5y - 6z + 9 = 0$

Pair of planes

The equation $H \equiv ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ represents a pair of planes passing through the origin

then $abc+2fgh-af^2-bg^2-ch^2=0$,

$f^2 \geq bc$, $g^2 \geq ac$, $h^2 \geq ab$.

$$ax^2+by^2+cz^2+2hxy+2fyz+2gzx+dx+ey+kz+s=0$$

$$ax^2+by^2+cz^2+2hxy+2fyz+2gzx=0$$

$$ax^2+(2hy+2gz)x+by^2+cz^2+2fyz=0$$

$$Ax^2+Bx+C=0$$

$A=a$ $\quad B=2hy+2gz$ $\quad C=by^2+cz^2+2fyz$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

① Show that the equation $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents a pair of planes. Also find the angle between them.

$$2x^2-6y^2-12z^2+18yz+2zx+xy=0$$

$$2x^2+(2z+y)x+(-6y^2-12z^2+18yz)=0$$

$A=2$, $B=2z+y$ $\quad C=(-6y^2-12z^2+18yz)$

$$B^2-4AC=(2z+y)^2-4(2)(-6y^2-12z^2+18yz)$$

$$=4z^2+y^2+4yz+48y^2+96z^2-144yz$$

$$=100z^2+49y^2-140yz$$

$$=10z^2+7y^2-2(10z)(7y)$$

$$B^2-4AC=(7y-10z)^2$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$= \frac{-(2z+y) \pm \sqrt{(7y-10z)^2}}{2(2)}$$

$$= \frac{-2z-y \pm (7y-10z)}{4}$$

$$x = \frac{-2z-y+7y-10z}{4}$$

$$4x = 6y-12z$$

$$4x - 6y+12z=0$$

$$2x-3y+6z=0 \quad \text{—— (1)}$$

$$x = \frac{-2z-y-7y+10z}{4}$$

$$4x = -2z-y-7y+10z$$

$$4x = 8z-8y$$

$$4x+8y-8z=0$$

$$x+2y-2z=0 \quad \text{—— (2)}$$

The required planes are

$$2x-3y+6z=0$$

$$x+2y-2z=0$$

$a_1=2 \quad b_1=-3 \quad c_1=6$

$a_2=1 \quad b_2=2 \quad c_2=-2$

angle between (1) and (2)

$$\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{|2(1)+(-3)2+6(-2)|}{\sqrt{2^2+(-3)^2+6^2}\sqrt{1^2+2^2+(-2)^2}}$$

$$= \frac{|2-6-12|}{\sqrt{4+9+36}\sqrt{1+4+4}}$$

$$= \frac{|-16|}{\sqrt{49}\sqrt{9}}$$

$$= \frac{16}{7(3)}$$

$$\cos\theta = \frac{16}{21}$$

$$\theta = \cos^{-1}\left(\frac{16}{21}\right)$$

2. Find the planes represented by the equation $x^2-2y^2-z^2-xy+3yz-6x+3y+9=0$ and hence find the angle between them

$$x^2-2y^2-z^2-xy+3yz-6x+3y+9=0$$

$$x^2+(-y)x+3yz-2y^2-z^2=0$$

$$A=1, \quad B=-y, \quad C=3yz-2y^2-z^2=0$$

$$B^2-4AC = (-y)^2-4(1)(3yz-2y^2-z^2)$$

$$= y^2-12yz+8y^2+4z^2$$

$$= 9y^2+4z^2-12yz$$

$$= (3y)^2+(2z)^2-2(3y)(2z)$$

$$= (3y-2z)^2$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$x = \frac{-(-y) \pm \sqrt{(3y-2z)^2}}{2(1)}$$

$$x = \frac{y \pm 3y-2z}{2}$$

$$2x = y \pm 3y-2z$$

$2x = y+3y-2z$ or $2x = y-(3y-2z)$

$2x-4y+2z=0$ or $2x+2y+2z=0$

$x-2y+z=0$ or $x+y-z=0$

Let the required planes are

$x-2y+z+d_1=0$ and $x+y-z+d_2=0$

Then

$(x-2y+z+d_1)(x+y-z+d_2) = x^2-2y^2-z^2-xy+3yz-6x+3y+9$

$x^2+xy-xz+xd_2-2xy-2y^2+2yz-2yd_2+xz+yz-z^2+zd_2+xd_1+yd_1-zd_1+d_1d_2 = x^2-2y^2-z^2-xy+3yz-6x+3y+9$

$x^2-2y^2-z^2-xy+3yz+(d_1+d_2)+(-2d_2+d_1)y+(-d_1+d_2)z+d_1d_2 = x^2-2y^2-z^2-xy+3yz-6x+3y+9$

$d_1+d_2=-6$

$-d_1-2d_2=3$

$-d_1+d_2=0$

$d_1d_2=0$

$d_2=d_1$

$d_1(d_1)=9$

$d_1^2=9 \Rightarrow d_1=\pm 3$

$d_1=-3$

$d_1+d_2=-6$

$-3+d_2=-6$

$d_2=-6+3$

$d_2=-3$

$d_1=-3$ and $d_2=-3$

$x-2y+z-3=0$

$x+y-z-3=0$

angle between two planes

$$\cos\theta = \frac{|1(1)+(-2)(1)+(1)(-1)|}{\sqrt{1^2+(-2)^2+(1)^2}\sqrt{(1)^2+1^2+(-1)^2}}$$

$$= \frac{|1-2-1|}{\sqrt{1+4+1}\sqrt{1+1+1}} = \frac{|-2|}{\sqrt{6}\sqrt{3}} = \frac{2}{\sqrt{18}}$$

$$= \frac{2}{\sqrt{9}\sqrt{2}} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}\sqrt{2}}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

(3) Show that $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$ represents a pair of parllel planes and find the distance between them.

The given plane is

$x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$

$x^2+4y^2+9z^2-12yz-6zx+4xy=0$

$x^2+(-6z+4y)x+4y^2+9z^2-12yz=0$

$A=1 \quad B=4y-6z \quad C=4y^2+9z^2-12yz$

$B^2-4AC = (4y-6z)^2 - 4(1)(4y^2+9z^2-12yz)=0$

$=16y^2+36z^2-48yz-16y^2-36z^2+48yz$

$=0$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$x = \frac{-(4y-6z) \pm \sqrt{0}}{2(1)}$$

$2x = -4y+6z$

$2x+4y-6z=0 \Rightarrow x+2y-3z=0$

Let the required plane is

$x+2y-3z+d_1=0$ or $x+2y-3z+d_2=0$

$(x+2y-3z+d_1)(x+2y-3z+d_2)=x^2+4y^2+9z^2-12yz-6xz+4xy+5x+10y-15z+6$

$x^2+2xy-3xz+xd_2+2xy+4y^2-6yz+2yd_2-3xz-6yz+9z^2-3zd_2+xd_1+2yd_1-3zd_1+d_1d_2=x^2+4y^2+9z^2-12yz-6x^2+4xy+5x+10y-15z+16$

$x^2+4y^2+9z^2-12yz-6xz+4xy+(d_1+d_2)x+$
$(2d_1+2d_2)y+(-3d_1-3d_2)z+d_1d_2 = x^2+4y^2+9z^2$
$-12yz-6xz+4xy+5x+10y-15z+6$

$d_1+d_2=5$
$2d_1+2d_2=10$
$-3d_1-3d_2=-15$
$d_1d_2=6$

$d_1+d_2=5$
$d_1=5-d_2$
$d_1d_2=6$
$(5-d_2)d_2=6$
$5d_2-d_2^2=6$
$d_2^2-5d_2+6=0$
$d_2^2-3d_2-2d_2+6=0$
$d_2(d_2-3)-2(d_2-3)=0$
$(d_2-3)(d_2-2)=0$
$d_2=3$ or $d_2=2$
$d_1+d_2=5$
$d_1+3=5$
$d_1=5-3$
$d_1=2$
$(d_1,d_2)=(2,3)$

The planes are
$x+2y-3z+2=0$ and $x+2y-3z+3=0$

Angle between two planes.

$$\cos\theta = \frac{|(1)(1)+(2)(2)+(-3)(-3)|}{\sqrt{1^2+2^2+(-3)^2}\sqrt{1^2+2^2+(-3)^2}}$$

$$= \frac{|1+4+9|}{\sqrt{1+4+9}\sqrt{1+4+9}} = \frac{|14|}{\sqrt{14}\sqrt{14}} = \frac{\sqrt{14}\cdot\sqrt{14}}{\sqrt{14}\sqrt{14}} = 1$$

$\theta = \cos^{-1}(1)$

$\theta = 0$

parllel planes

distance between parllel planes

$$\frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|2-3|}{\sqrt{1^2+2^2+(-3)^2}} = \frac{|-1|}{\sqrt{1+4+9}} = \frac{1}{\sqrt{14}}$$

2. Lines

1. The lines are divided into two types

i. symmetric form - $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

ii. unsymmetric form - $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$

2. The equation of a line passing through two points (x_1, y_1, z_1) & (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

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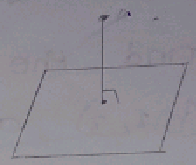
① Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$.

Imp



The dr's of the line that is perpendicular to the plane are

$a = 2, b = -1, c = 1$

The line that passes through (1, 3, 4) and having dr's 2, -1, 1 is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = k$$

$$\frac{x-1}{2} = k, \quad \frac{y-3}{-1} = k, \quad \frac{z-4}{1} = k$$

$$x - 1 = 2k, \quad y - 3 = -k, \quad z - 4 = k$$

$$x = 2k + 1, \quad y = -k + 3, \quad z = k + 4$$

If this is on $2x - y + z + 3 = 0$

$2(2k+1) - (-k+3) + (k+4) + 3 = 0$

$4k + 2 + k - 3 + k + 7 = 0$

$6k + 6 = 0$

$6k = -6$

$k = -6/6$

$k = -1$

$x_0 = -1$, $y_0 = 4$, $z_0 = 3$

Mid point of $PP' = k\left[\frac{x_1 + x_1'}{2} = x_0 , \frac{y_1+y_1'}{2} = y_0 , \frac{z_1+z_1'}{2} = z_0\right]$

image is $x_1' = 2x_0 - x_1 \Rightarrow x_1' = 2(-1) - 1 = -2-1 = -3$

$y_1' = 2y_0 - y_1 \Rightarrow y_1' = 2(4) - 3 = 8-3 = 5$

$z_1' = 2z_0 - z_1 \Rightarrow z_1' = 2(3) - 4 = 6-4 = 2$

The image $P = (-3, 5, 2)$

② Find the equation of the line through (1, 2, 3) and parallel to the line $x - y + 2z = 5$, $3x + y + z = 6$.

Imp

Sol:- The given parallel to the lines are

$x - y + 2z = 5$, $3x + y + z = 6$

The normal vector to the plane

$x - y + 2z = 5$ is $a = (1, -1, 2)$

The normal vector to the plane

$3x + y + z = 6$ is $b = (3, 1, 1)$

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$= i(-1-2) - j(1-6) + k(1-3)$

$= -3i + 5j + 4k$

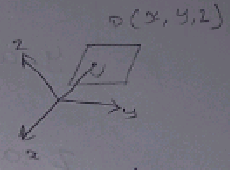
∴ The direction ratio's of the given line $= (-3, 5, 4)$

The equation to the required line are

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

10.B (3) A variable plane makes intercepts on the coordinate axes, the sum of whose squares is k^2 (a constant). Show that the
Imp locus of the foot of the ⊥r from the origin to the plane is $(x^{-2}+y^{-2}+z^{-2})(x^2+y^2+z^2)^2 = k^2$

Sol: Let the foot of the ⊥r of the origin (0,0,0) on variable plane is (x, y, z). Then



dr's to the normal of the variable plane are

$$l = x_1 - 0, \quad m = y_1 - 0, \quad n = z_1 - 0$$

Then the required plane is of the form.

$$lx + my + nz + d = 0 \longrightarrow (1)$$

$$x_1 x + y_1 y + z_1 z + d = 0$$

Since $P(x_1, y_1, z_1)$ is on (1)

$$x_1 x_1 + y_1 y_1 + z_1 z_1 + d = 0$$

$$x_1^2 + y_1^2 + z_1^2 + d = 0$$

$$d = -(x_1^2 + y_1^2 + z_1^2)$$

The required plane is

$$x_1 x + y_1 y + z_1 z + d = 0$$

$$x_1 x + y_1 y + z_1 z + (-(x_1^2 + y_1^2 + z_1^2)) = 0$$

$$x_1 x + y_1 y + z_1 z = x_1^2 + y_1^2 + z_1^2$$

$$x_1 x + y_1 y + z_1 z$$ dividing $x_1^2 + y_1^2 + z_1^2$ on both sides

$$\frac{x_1 x}{x_1^2+y_1^2+z_1^2} + \frac{y_1 y}{x_1^2+y_1^2+z_1^2} + \frac{z_1 z}{x_1^2+y_1^2+z_1^2} = 1$$

$$\frac{x}{\frac{x_1^2+y_1^2+z_1^2}{x_1}} + \frac{y}{\frac{x_1^2+y_1^2+z_1^2}{y_1}} + \frac{z}{\frac{x_1^2+y_1^2+z_1^2}{z_1}} = 1$$

It is in the form of $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

This plane meets

x-axis at $a=\frac{x_1^2+y_1^2+z_1^2}{x_1}$

y-axis at $b=\frac{x_1^2+y_1^2+z_1^2}{y_1}$

z-axis at $c=\frac{x_1^2+y_1^2+z_1^2}{z_1}$

It is given that

$a^2+b^2+c^2=k^2$

$$\left(\frac{x_1^2+y_1^2+z_1^2}{x_1}\right)^2+\left(\frac{x_1^2+y_1^2+z_1^2}{y_1}\right)^2+\left(\frac{x_1^2+y_1^2+z_1^2}{z_1}\right)^2=k^2$$

$$\frac{(x_1^2+y_1^2+z_1^2)^2}{x_1^2}+\frac{(x_1^2+y_1^2+z_1^2)^2}{y_1^2}+\frac{(x_1^2+y_1^2+z_1^2)^2}{z_1^2}=k^2$$

$$(x_1^2+y_1^2+z_1^2)^2\left(\frac{1}{x_1^2}+\frac{1}{y_1^2}+\frac{1}{z_1^2}\right)=k^2$$

$$(x_1^2+y_1^2+z_1^2)^2(x_1^{-2}+y_1^{-2}+z_1^{-2})=k^2$$

The locus of foot of the $\perp$r is

$$(x_1^{-2}+y_1^{-2}+z_1^{-2})(x_1^2+y_1^2+z_1^2)^2=k^2$$

(4) Find the image of the point $(2,-1,3)$ in the plane $3x-2y-z=9$.

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Sol: The Given point $P=(2,-1,3)$ (x_1, y_1, z_1) and plane is $3x-2y-z=9$ ——— (1)

Let $Q(x_1,y_1,z_1)$ be the image of the point P.

The dr's of the line that is perpendicular to the plane are

$a=3 \quad b=-2 \quad c=-1$ is

$$\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-3}{-1}=k$$

$$\frac{x-2}{3}=k \quad \frac{y+1}{-2}=k \quad \frac{z-3}{-1}=k$$

$$x=3k+2 \quad y=-2k-1 \quad z=-k+3$$

If this is on $3x-2y-z=9$

$$3(3k+2)-2(-2k-1)-(-k+3)-9=0$$

$$9k+6+4k+2+k-3-9=0$$

$$14k-4=0$$

$$14k=4$$

$$k=4/14$$

$$k=2/7$$

$$x=3(2/7)+2 \quad , \quad y=-2(2/7)-1 \quad z=\frac{-2}{7}+3$$

$$=\frac{6+14}{7} \quad =\frac{-4-7}{7} \quad =\frac{-2+21}{7}$$

$$x_0=\frac{20}{7} \quad y_0=\frac{-11}{7} \quad z_0=\frac{19}{7}$$

mid point of $PP' = k\left[\frac{x_1+x_1'}{2}=x_0, \frac{y_1+y_1'}{2}=y_0, \frac{z_1+z_1'}{2}=z_0\right]$

The image is $x_1'=2x_0-x_1 \Rightarrow x_1'=2\left(\frac{20}{7}\right)-2$

$$x_1'=\frac{40-14}{7} \Rightarrow x_1'=\frac{26}{7}$$

$$y_1'=2y_0-y_1 \Rightarrow y_1'=2(-11/7)-(-1) \Rightarrow y_1'=\frac{-22+7}{7}$$

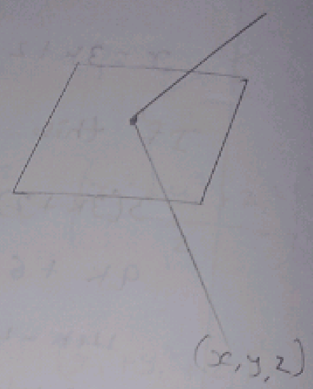
$$y_1'=\frac{-15}{7}$$

$z_1' = 2z_0 - z_1 \Rightarrow z_1' = 2\left(\frac{19}{2}\right) - 3 \Rightarrow z_1' = \frac{38-21}{7}$

$z_1' = \frac{17}{7}$

The image $P = \frac{26}{7}, -\frac{15}{7}, \frac{17}{7}$

⑤ Find the image of the line $\frac{x-1}{9} = \frac{y-2}{1} = \frac{z+3}{-3}$ in the plane $3x-3y+10z-26=0$



Sol: The given line is

$L: \frac{x-1}{9} = \frac{y-2}{1} = \frac{z+3}{-3} = k_1$

Given plane is

$\pi: 3x - 3y + 10z - 26 = 0$

General point on L is of the form

$x = 9k_1 + 1$
$y = k_1 + 2$
$z = -k_1 - 3$

If this point is on π

$3(9k_1+1) - 3(k_1+2) + 10(-k_1-3) - 26 = 0$

$27k_1 + 3 - 3k_1 - 6 - 10k_1 - 30 - 26 = 0$

$14k_1 - 59 = 0$

$14k_1 = 59$

$k_1 = 59/14$

$x_2 = 9k_1 + 1 \Rightarrow 9\left(\frac{59}{14}\right) + 1 \Rightarrow \frac{545}{14}$

$y_2 = k_1 + 2 \Rightarrow \frac{59}{14} + 2 \Rightarrow \frac{59+28}{14} \Rightarrow \frac{87}{14}$

$z_2 = -k_1 - 3 \Rightarrow -\frac{59}{14} - 3 \Rightarrow \frac{-59-42}{14} \Rightarrow \frac{-10}{14}$

The equations of the line ⊥r to π and passing through (1, 2, -3) is
$x_1\ y_1\ z_1$

$$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z-(-3)}{10} = k_2$$

$$\frac{x-1}{3} = k_2, \quad \frac{y-2}{-3} = k_2, \quad \frac{z+3}{10} = k_2$$

$$x = 3k_2 + 1, \quad y = -3k_2 + 2, \quad z = 10k_2 - 3$$

If this is on π

$$3(3k_2+1) - 3(-3k_2+2) + 10(10k_2-3) - 26 = 0$$

$$9k_2 + 3 + 9k_2 - 6 + 100k_2 - 30 - 26 = 0$$

$$118k_2 - 59 = 0$$

$$k_2 = \frac{59}{118}$$

$$= 1/2$$

$$x_0 = 3(1/2) + 1 \Rightarrow 5/2$$

$$y_0 = -3(1/2) + 2 \Rightarrow 1/2$$

$$z_0 = 10(1/2) - 3 \Rightarrow 2$$

$$x_1' = 2x_0 - x_1 \Rightarrow x_1' = 2\left(\frac{5}{2}\right) - 1 \Rightarrow x_1' = \frac{10-2}{2} \Rightarrow \frac{8}{2}$$

$$x_1' = 4$$

$$y_1' = 2y_0 - y_1 \Rightarrow y_1' = 2(1/2) - 2 \Rightarrow y_1' = \frac{2-4}{2} \Rightarrow y_1' = -1$$

$$z_1' = 2z_0 - z_1 \Rightarrow z_1' = 2(2) - (-3) \Rightarrow z_1' = 4 + 3 \Rightarrow z_1' = 7$$

$$(x_1', y_1', z_1') = (4, -1, 7)$$

The image of the line is

$$\frac{x-4}{\frac{548}{14} - 4} = \frac{y-(-1)}{\frac{87}{14} - (-1)} = \frac{z-7}{\frac{-101}{14} - 7} = k_3$$

$$\frac{x-4}{\frac{493}{14}} = \frac{y+1}{\frac{101}{14}} = \frac{z-7}{\frac{-199}{14}} \quad (\text{or}) \quad \frac{x-4}{493} = \frac{y+1}{101} = \frac{z-7}{-199}$$

Coplanar Lines

(1) Imp Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$;

$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Poo point of intersection and the plane containing the lines.

Sol: L_1: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ —— (1)

L_2: $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ —— (2)

Let the point on (1) (1, 2, 3) & points the point on (2) is (2, 3, 4)

Let the dr's of (1) (2, 3, 4) & dr's of (2) is (3, 4, 5)

To prove that (1) & (2) are coplanar we have P.T (1) and (2) are neither ⊥r nor parallel

Now
$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$$\begin{vmatrix} 1-2 & 2-3 & 3-4 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$\begin{vmatrix} -1 & -1 & -1 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$-1(15-16) + 1(10-12) - 1(8-9)$

$1 - 2 + 1 = 0$

from equation (1) $\frac{x-1}{2} \times \frac{y-2}{3}$

$3(x-1) = 2(y-2)$

$3x-3 = 2y-4$

$3x-2y-3+4=0$

$3x-2y+1=0$ —— ③

from eq ① $\frac{y-2}{3} = \frac{z-3}{4} \Rightarrow 4(y-2) = 3(z-3)$

$\Rightarrow 4y-8 = 3z-9$

$4y-3z-8+9=0$

$4y-3z+1=0$ —— ④

from eq ② $\frac{x-2}{3} = \frac{y-3}{4} \Rightarrow 4(x-2) = 3(y-3)$

$\Rightarrow 4x-3y+1=0$ —— ⑤

Solving ③ & ⑤ $\Rightarrow$ ③ $\times 3 \Rightarrow 9x-6y+3=0$

⑤ $\times 2 \Rightarrow 8x-6y+2=0$

$x+1=0$

$x=-1$

③ $\Rightarrow 3x-2y+1=0$

$3(-1)-2y+1=0$

$-3-2y+1=0$

$-2=2y$

$y=-2/2$

$y=-1$

④ $\Rightarrow 4y-3z+1=0$

$4(-1)-3z+1=0$

$-4+1=3z$

$-\frac{3}{3}=z$

$z=-1$

∴ lines of ① & ② intersects at $(-1,-1,-1)$

Let the eq of a plane through lines ① & ② is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

$(x-1)(15-16) - (y-2)(10-13) + (z-3)(8-9) = 0$

$(x-1)(-1) + (y+2)(-2) + (z-3)(-1) = 0$

$-x+1+2y-4-z+3=0$

$-x+2y-z=0$

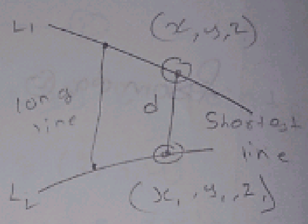
$x-2y+z=0$

Equations of shortest distance line

$L_1: \frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} = k_1$

$L_2: \frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2} = k_2$

$$\begin{vmatrix} l & m & n \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$



$$\frac{l}{m_1n_2-m_2n_1} = \frac{m}{l_2n_1-l_1n_2} = \frac{n}{l_1m_2-l_2m_1}$$

$l = m_1n_2-m_2n_1$, $m = l_2n_1-l_1n_2$, $n = l_1m_2-l_2m_1$

Shortest distance

$a = x_2-x_1$, $b = y_2-y_1$, $Z = Z_2-Z_1$

$\bar{A} = a\hat{i}+b\hat{j}+c\hat{k}$, $\bar{B} = l\hat{i}+m\hat{j}+n\hat{k}$

Shortest distance = projection of $\bar{A}$ on $\bar{B}$

$$d = \frac{|\bar{A}\cdot\bar{B}|}{|\bar{B}|}$$

$$= \frac{|al+bm+cn|}{\sqrt{l^2+m^2+n^2}}$$

Equations of SD line

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0 = \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix}$$

point of intersection between SD line and $L_1 L_2$

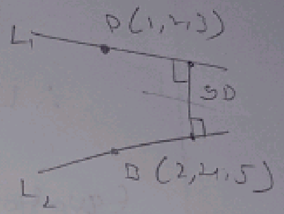
General point on L_1

$x = k_1 l_1 + x_1$, $y = k_1 m_1 + y_1$, $z = k_2 n_2 + z_2$

Substitute this in π_1 to get k_2

① Find the length and equations of the S.D line between the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$



Let l, m, n be dr's of SD line

$$\begin{vmatrix} l & m & n \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\frac{l}{15-16} = \frac{m}{12-10} = \frac{n}{8-9}$

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$\frac{l}{-1} = \frac{m}{2} = \frac{n}{-1}$

$l = -1$, $m = 2$, $n = -1$

Dr's of $\overline{PQ}$

$a = 2-1$ $\quad b = 4-2$ $\quad c = 5-3$

$b = 2$ $\quad c = 2$

$a = 1$

$(a, b, c) = (1, 2, 2)$

$\bar{A} = a\hat{i} + b\hat{j} + c\hat{k} \Rightarrow 1\hat{i} + 2\hat{j} + 2\hat{k}$

$\bar{B} = l\hat{i} + m\hat{j} + n\hat{k} \Rightarrow -1\hat{i} + 2\hat{j} - 1\hat{k}$

SD = projection of $\bar{A}$ and $\bar{B}$

$$= \frac{|\bar{A}\cdot\bar{B}|}{|\bar{B}|}$$

$$= \frac{|al+bm+cn|}{\sqrt{l^2+m^2+n^2}}$$

$$= \frac{|1(-1)+2(2)+2(-1)|}{\sqrt{(-1)^2+2^2+(-1)^2}}$$

$$= \frac{|-1+4-2|}{1+4+1}$$

$\therefore$ Length $= \frac{1}{\sqrt{6}}$

Equation of SD line

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ -1 & 2 & -1 \end{vmatrix} = 0 = \begin{vmatrix} x-2 & y-4 & z-5 \\ 3 & 4 & 3 \\ -1 & 2 & -1 \end{vmatrix}$$

$(x-1)(-3-8)-(y-2)(-2+4)+(z-3)(4+3)=0=(x-2)$
$(-4-10)-(y-4)(-3+5)+(z-5)(6+4)$

$\Rightarrow (x-1)(-11)+(-y+2)(2)+(z-3)(7)=0=(x-2)(-14)$
$+(-y+4)(+2)+(z-5)(10)$

$\Rightarrow -11x+11-2y+4+7z-21=0=-14x+28-2y+8$
$+10z-50$

$\Rightarrow -11x-2y+7z-6=0=-14x-2y+10z-14$

$(-1) \qquad\qquad\qquad\qquad (-2)$

$\Rightarrow 11x+2y-7z+6=0=7x+y-5z+7$

$\pi_1 \quad = 0 = \pi_2$

$\pi_1 = 11x+2y-7z+6$

$\pi_2 = 7x+y-5z+7$

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = k_1$

$\frac{x-1}{2} = k_1 \quad \frac{y-2}{3} = k_1 \quad \frac{z-3}{4} = k_1$

$x-1 = 2k_1 \quad y-2 = 3k_1 \quad z-3 = 4k_1$

$x = 2k_1+1 \quad y = 3k_1+2 \quad z = 4k_1+3$

Substitute (x, y, z) in π_2

$\pi_2 : 7x+y-5z+7=0$

$7(2k_1+1)+(3k_1+2) -5(4k_1+3)+7=0$

$14k_1+7+3k_1+2-20k_1-15+7=0$

$-3k_1+1=0$

$1=3k_1$

$k_1 = 1/3$

$x = 2k_1+1 \quad y = 3k_1+2 \quad z = 4k_1+3$

$x = 2(1/3)+1 \quad y = 3(1/3)+2 \quad z = 4(1/3)+3$

$x = \frac{2+3}{3} \quad y = \frac{3+6}{3} \quad z = \frac{4+9}{3}$

$x = \frac{5}{3} \quad y = \frac{9}{3} \quad z = \frac{13}{3}$

$x = \frac{5}{3} \quad y = 3 \quad z = \frac{13}{3}$

$\pi_1 = 11x+2y-7z+6$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} = k_2$

$\frac{x-2}{3} = k_2 \quad \frac{y-4}{4} = k_2 \quad \frac{z-5}{5} = k_2$

$x = 3k_2+2 \quad y = 4k_2+4 \quad z = 5k_2+5$

Substitute (x, y, z) in π_1

$\pi : 11x+2y-7z+6=0$

$11(3k_2+2)+2(4k_2+4)-7(5k_2+5)+6=0$

$33k_2 + 22 + 8k_2 + 8 - 35k_2 - 35 + 6 = 0$

$6k_2 + 1 = 0$

$k_2 = -1/6$

$x = 3(-1/6) + 2 \quad y = 4(-1/6) + 4 \quad z = 5(-1/6) + 5$

$= \frac{-3+12}{6} \quad = \frac{-4+24}{6} \quad = \frac{-5+30}{6}$

$= \frac{9}{6} \quad = \frac{20}{6} \quad = \frac{25}{6}$

$= 3/2 \quad = 10/3 \quad = \frac{25}{6}$

$(x_1, y_1, z_1) = \left(3/2, 10/3, \frac{25}{6}\right)$

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(2) Find the S.D and the equations of S.D line between the lines

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$; $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Also find the points in which the S.D line meets the given lines.

Given lines are

$L_1: \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ —— (1)

$L_2: \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ —— (2)

Let the eq of the plane through line (1) & ||el to line (2)

$(x_1, y_1, z_1) = (3, 8, 3)$

$(x_2, y_2, z_2) = (-3, -7, 6)$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-3 & y-8 & z-3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = 0$$

$(x-3)(-4-2) - (y-8)(12+3) + (z-3)(6-3)$

$(x-3)(-6) + (-y+8)(+15) + (z-3)3 = 0$

$-6x + 18 - 15y + 120 + 3z - 9 = 0$

$-6x - 15y + 3z + 129 = 0$

$-3(2x + 5y - z - 43) = 0$

$A : 2x + 5y - z - 43 = 0$

a point on line ① is

Now S.D is

$(-3, -7, 6)$ and plane from line ①

SD = projection of $\bar{B}$ on $\bar{A}$ $\Rightarrow \dfrac{|\bar{A}\cdot\bar{B}|}{|\bar{B}|}$

$$= \frac{|a(l_2) + b(m_2) + c(n_2)|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|2(-3) + 5(-7) - 6 - 43|}{\sqrt{2^2 + 5^2 + (-1)^2}}$$

$$= \frac{|-6 - 35 - 6 - 43|}{\sqrt{4 + 25 + 1}}$$

$$= \frac{|-90|}{\sqrt{30}} \Rightarrow \frac{90}{\sqrt{30}} \Rightarrow \frac{3 \times 30}{\sqrt{30}} \Rightarrow 3\sqrt{30}$$

Let the equation of the plane through line ② & parallel to line ①

$$\begin{vmatrix} x+3 & y+7 & z-6 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = 0$$

$(x+3)(-4-2) - (y+7)(12+3) + (z-6)(6-3) = 0$

$(x+3)(-6) - (y+7)(15) + (z-6)(3) = 0$

$-6x - 18 - 15y - 105 + 3z - 18 = 0$

$-6x - 15y + 3z - 141 = 0$

$-3(2x + 5y - z + 47) = 0$

$2x + 5y - z + 47 = 0$

Equation of lines of S.D are

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0 = \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\begin{vmatrix} x-3 & y-8 & z-3 \\ 3 & -1 & 1 \\ 2 & 5 & -1 \end{vmatrix} = 0 = \begin{vmatrix} x+3 & y+7 & z-6 \\ -3 & 2 & 4 \\ 2 & 5 & -1 \end{vmatrix}$$

$\Rightarrow (x-3)(1-5) - (y-8)(-3-2) + (z-3)(15+2) = 0 = (x+3)(-2-20) - (y+7)(3-8) + (z-6)(-15-4)$

~~$(-18x-14) - 4(x\_3)$~~

$\Rightarrow -4(x-3) + 5(y-8) + 17(z-3) = 0 = -22(x+3) + 5(y+7) - 19(z-6)$

$\Rightarrow -4x + 12 + 5y - 40 + 17z - 51 = 0 = -22x - 66 + 5y + 35 - 19z + 114$

$-4x+5y+17z-79=0 = -22x+5y-19z+83$

$4x-5y-17z+79=0 = 22x-5y+19z-83$

③ Find the image of the line $\frac{x-1}{9}=\frac{y-2}{1}=\frac{z+3}{-3}$ in the plane $3x-3y+10z-26=0$

Given line

$L: \frac{x-1}{9} = \frac{y-2}{1} = \frac{z+3}{-3}$ ——— ①

and plane $\pi: 3x-3y+10z-26=0$ — ②

$① \Rightarrow \frac{x-1}{9} = \frac{y-2}{1} = \frac{z+3}{-3} = k_1$

$\frac{x-1}{9} = k_1, \quad \frac{y-2}{1} = k_1 \quad \frac{z+3}{-3} = k_1$

$x = 9k_1+1 \quad y = k_1+2 \quad z = -3k_1-3$

Substitue x, y, z in the plane

$3x-3y+10z-26=0$

$3(9k_1+1) - 3(k_1+2) + 10(-3k_1-3) - 26 = 0$

$27k_1+3-3k_1-6-30k_1-30-26=0$

$27k_1-3k_1-30k_1-59=0$

$-6k_1-59=0$

$-59=6k_1$

$k_1 = \frac{-59}{6}$

$\therefore [9k_1+1, k_1+2\}, -3k_1-3]$

$\therefore \left(9\left(-\frac{59}{6}\right)+1, -\frac{59}{6}+2, -3\left(-\frac{59}{6}\right)+3\right)$

$\left[\frac{525}{6}, -\frac{47}{6}, \frac{159}{6}\right]$ through $(1,2,-3)$

Now the equation to L_2 of the plane is and having dr's

$$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z-(-3)}{10} = k_2$$

$x = 3k_2+1$ $\quad y = -3k_2+2$ $\quad z = 10k_2-3$

$\therefore$ ② $\Rightarrow 3x-3y+10z-26=0$

$3(3k_2+1)-3(-3k_1+2)+10(10k_2-3)-26=0$

$9k_2+3+9k_1-6+100k_2-30-26=0$

$118k_2-59=0$

$118 \cdot k_2 = 59$

$k_2 = 59/118$

$k_2 = 1/2$

$\therefore [3k_2+1, -3k_2+2, 10k_2-3]$

$[3(1/2)+1, -3(1/2)+2, 10(1/2)-3]$

$[5/2, 1/2, 2)$

Let $p(x_1, y_1, z_1)$ be the image of $(1, 2, -3)$

$\therefore$ mid point $[5/2, 1/2, 2] = \left[\frac{x_1+1}{2}, \frac{y_1+2}{2}, \frac{z_1-3}{2}\right]$

$\frac{5}{2} = \frac{x_1+1}{2}$, $\quad \frac{1}{2} = \frac{y_1+2}{2}$ $\quad 2 = \frac{z_1-3}{2}$

$x_1 = 5-1$ $\quad y_1 = 1-2$ $\quad z_1 = 4+3$

$x_1 = 4$ $\quad y_1 = -1$ $\quad z_1 = 7$

$(x_1, y_1, z_1) = (4, -1, 7)$

$\therefore$ The eq. of the line through $(4,-1,7)$ & $\left(-\frac{525}{6}, -\frac{47}{0}, \frac{159}{0}\right)$ is

$$\frac{x-4}{-\frac{525}{6}-4} = \frac{y-(-1)}{-\frac{47}{6}+1} = \frac{z-7}{\frac{159}{6}-7}$$

$$\frac{x-4}{-549} = \frac{y+1}{-41} = \frac{z-7}{117}$$

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